DELAY–DEPENDENT STABILIZATION OF INPUT–DELAYED SYSTEMS BY LINEAR CONTROL: A NEW DESIGN METHODOLOGY

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In this paper, a new design approach based on Lagrange mean value theorem is used for the first time for the stabilization of multivariable input-delayed system by linear controller. The delay–dependent asymptotical stability conditions are derived by using augmented Lyapunov-Krasovskii functionals and formulated in terms of conventional Lyapunov matrix equations and some simple matrix inequalities. Proposed design approach is extended to robust stabilization of multi-variable input delayed systems with unmatched parameter uncertainties. The maximum upper bound of delay size is computed by using simple optimization algorithm. A liquid monopropellant rocket motor with a pressure feeding system is considered as a numerical design example. Design example shows the effectiveness of our proposed design approach. The paper has a great potential in the stability analysis of time delay systems and design of time-delay controllers and may open a new direction in this area.

Keywords: input delayed systems, Lagrange mean value theorem, robust stabilization, delay-dependent stability.

1. INTRODUCTION

Time-delay effect is frequently encountered in oil-chemical systems, metallurgy and machine-tool process control, nuclear reactors, bio-technical systems missile-guidance and aircraft control systems, aerospace remote control and communication control systems, etc. The presence of delay effect complicates the analysis and design of control systems. Moreover, time-delay effects in the state vector, especially in the control input degrade the control performances and make the closed–loop stabilization problem challenging. For better understanding of time-delay effect properties let us briefly analyze the existing design methodologies. There are three basic control design methodologies for the stabilization of input delayed systems: 1) Smith predictor method, 2) Reduction method, 3) Memoryless control approach.

A common design method of input-delayed systems is well known Smith predictor control to cancel the effect of time-delay. Smith predictor is a popular and very effective long delay compensator for stable processes. The main advantage of the Smith predictor control method is that, the time-delay is eliminated from the characteristic equation of the closed-loop system. Classical Smith predictor was suggested by Smith [1], [2]. Modified Smith predictor scheme's have been advanced by Marshall [3], Aleviskas and Seborg [4], Watanabe and Ito [5], Watanabe, Ishiyama and Ito [6], Al-Sunni and Al-Nemer [7], Majhi and Atherton [8], etc.

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Note that Smith Predictor removes only the time–delay from closed–loop while it is remained in feed-forward path. Therefore, it is also an input-delayed system. An extension of the Smith predictor method for the MIMO systems with state and input delays is considered by Alevisaxis and Seborg [4]. The control algorithm in a Smith Predictor is normally a PI-controller. The D–part normally is not used since the prediction is performed by the dead–time compensation. Prediction through derivation is not suitable when the process contains a long dead– time. Replacing a PID-controller with a Smith predictor gives a drastic increase in operational complexity. This is the main reason why most processes with long time–delay are still controlled by PI-controllers. A modified Smith predictor based on industrial PI-controller is designed by Hagglund [9]. A modified Smith predictor and controller for unstable processes with time–delay are developed by DePaor [10]. Modified Smith predictor control for multivariable systems with delays and unmeasurable disturbances is extended by Watanabe, Ishiyama and Ito [6]. Modified Smith predictor and controller design procedure for unstable processes is proposed by Majhi and Atherton [8]. A Smith predictor fuzzy logic based PI-controller design for processes with long dead–time is proposed by Al-Sunni and Al-Nemer [7].

The second important control design method of input–delayed systems is the reduction method that was suggested by Kwon and Pearson [11].

This control strategy has been shown to overcome some of inherent problems of the conventional Smith predictor method. For example, unstable system can be stabilized and the effects of the initial conditions are taken into consideration. The reduction method, however, suffers from a weakness that the complete reduction to a delay free system is only possible with an exact model of the system. Reduction method is extended to time–varying system with distributed delays by Arstein [12]. A new robust stabilizing controller for multiple input–delayed system with parametric uncertainties by using a modified reduction method is proposed by Moon, Park and Kwon [13]. However, an industrial implementation of reduction method controllers is much complicated than conventional method.

The third design approach to stabilization of input-delayed systems is so-called memoryless control method, which is similar to the conventional linear control method. Such controllers have feedback of the current state only, are designed to delay–independent stabilization of input–delayed systems by using Lyapunov–Krasovskii functional method, for example, see Choi and Chung [14], Kim, Jeung and Park [15], Su, Chu and Wang [16], etc. However, this approach is conservative when the actual size of the delay is small. In fact, information on the size of the delay is often available in many processes. Hence, by using delay information and past control history as well as the current state delay–dependent controllers may provide much better performance than memoryless controllers.

In analysis and design of time-delay systems, in general, the Lyapunov-Krasovskii functional method is commonly used. Recent advances in time-delay systems are presented by Richard [17], Fridman and Shaked [18], Jafarov [19], Niculescu and Gu [20], Niculescu [21], Mahmoud [22], Gu, Kharitonov and Chen [23], Boukas and Liu [24]. Some sufficient delay-dependent stability conditions for linear delay perturbed systems are derived using exact Lyapunov-Krasovskii functionals by Kharitonov and Niculescu [25]. Several new LMI delay-dependent robust stability results for linear time-delay systems with unknown time-invariant delays by using Padé approximation are presented by Zhang, Knospe and Tsiotras [26]. Both delay-independent and delay-dependent robust stability LMI's from conditions for linear time-delay systems with unknown delays by using appropriately selected Lyapunov-Krasovskii functionals are systematically investigated by Zhang, Knospe and Tsiotras in another paper [27]. Stability of the internet network rate control with diverse delays based on Nyquist criterion is considered by Tian and Yang [28]. Improved delay-dependent stability conditions for time-delay systems in terms of strict LMI's avoiding cross terms are developed by Xu and Lam [29]. A new state transformation is introduced to exhibit the delay-dependent stability condition for time-delay systems by Mahmoud and Ismail [30]. Determining controllable sets from a time-delay description is given by Rhodes and Morari [31].

Resuming the brief analysis of references concerning the existing design approaches, it can be concluded that time-delay systems are intensively investigated recently by researchers in light of the above mentioned three directions. However, a new direction to stability analysis and controller design of time-delay systems is not developed. In this paper, we have attempted to present for discussion a principle new design approach to analysis and design of time-delay systems. Introduced new design approach may open a new direction in this field. Proposed design approach based on Lagrange mean value theorem is used for the first time for the stabilization of multivariable input delayed system. The delay–dependent asymptotical stability conditions are derived by using augmented Lyapunov-Krasovskii functionals and formulated in terms of conventional Lyapunov matrix equations and some simple matrix inequalities. Proposed design approach is extended to robust stabilization of multi-variable input delayed systems with unmatched parameter uncertainties. The maximum upper bound of delay size is computed by using simple optimization algorithm. A liquid monopropellant rocket motor with a pressure feeding system is considered as a numerical design example. Design example shows the effectiveness of our proposed design approach.

2. DELAY–DEPENDENT STABILIZATION BY LINEAR CONTROLLER

Let us consider the following control input-delayed multivariable system of the form:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t-h), \quad t > 0 \\ u(t) &= \phi(t), \qquad -h \le t \le 0 \end{aligned} \tag{1}$$

where $x(t)$ is the measurable state n-vector, $u(t)$ is the control input m–vector, in general $m \le n$, A is the unstable $(n \times n)$ - matrix and B is the $(n \times m)$ - matrix of full rank, $\phi(t)$ is known initial control function on interval $[-h, 0]$, $h > 0$ is a constant time–delay. If we consider a case where h is an unknown, then we assume that h is bounded $0 \le h < \bar{h}$.

We assume that time–delay system is completely state controllable [32]. This design method is based on the Lagrange mean value theorem familiar from classical Calculus [33], [34]. Remember that Lagrange mean value theorem is stated as follows:

$$\frac{f(b) - f(a)}{b - a} = f'(\xi), \quad a < \xi < b \tag{2}$$

where $f(x)$ is a continuous function at every point of the closed interval [a, b] and differentiable at every point of its interior (a, b) or in terms of delayed control input

$$u(t-h) = u(t) - h\dot{u}(\theta) \tag{3}$$

where θ is a point in $t - h < \theta < t$.

After introducing the θ parameter, the constructive delay-dependent asymptotical stability and robustly asymptotically stable conditions can be derived by using the augmented Lyapunov-Krasovskii functionals.

Now, after preparing the necessary background we can present a new continuous control design methodology for input-delayed systems with known or unknown but bounded delays. Substituting (3) into (1), the input-delayed system can be transformed to following system:

$$\dot{x}(t) = Ax(t) + Bu(t) - Bh\dot{u}(\theta) \tag{4}$$

Now we can utilize the conventional linear controller

$$u(t) = -Gx(t) \tag{5}$$

where G is the gain $(m \times n)$ matrix to be selected. Then,

$$\dot{x}(t) = (A - BG)x(t) + hBG\dot{x}(\theta) \tag{6}$$

where

$$\dot{x}(\theta) = Ax(\theta) + Bu(\theta - h) = Ax(\theta) - BGx(\theta - h) \tag{7}$$

since

$$u(\theta - h) = -Gx(\theta - h) \tag{8}$$

Hence, we have the following transformed state delayed multivariable system

$$\dot{x}(t) = \overline{A}x(t) + hBGAx(\theta) - h(BG)^2 x(\theta - h) \tag{9}$$

where $\overline{A} = A - BG$. G can be selected for example by pole placement, such that $\overline{A}$ has desirable eigenvalues.

As seen from (9), the state equation depends on: 1) current state $x(t)$, 2) near past history of the state $x(\theta)$, 3) far past history of the state $x(\theta - h)$.

Therefore, the problem of stabilization of system (9) is not simple. Now, we need to make the following assumption.

ASSUMPTION 1: Time-delay parameter θ is a time-dependent function and norm-bounded such that

$$0 < 1 - \eta \le \dot{\theta}(t) \le \eta < 1 \tag{10}$$

where η is a scalar.

Note that Assumption 1 is conventional and is commonly used by many authors, for example, by Ikeda and Ashida [35], Su and Chu [36], Su, Ji and Chu [37], Wu, He, She and Liu [38], Kim [39] etc.

We shall treat η as an adjustable parameter at the disposal of the designer. Therefore, we will have the freedom to change η in a way to produce satisfactory system performance. A useful feature that will become clear in the numerical example.

Stability results for transformed time-delay system (9) by using augmented Lyapunov–Krasovskii functionals can be formulated as follows.

THEOREM 1: Suppose that Assumption 1 holds. Then the transformed time-delay system (9) driven by linear controller (5) is delay-dependent asymptotically stable, if there are design parameters G and positive definite symmetric matrices P, R and T such that the following conditions are satisfied:

$$H = \begin{bmatrix} -Q & hD & -hC & 0 \\ hD^T & \eta R - (1-\eta)S & 0 & 0 \\ -hC^T & 0 & -(1-\eta)R & 0 \\ 0 & 0 & 0 & -T \end{bmatrix} < 0 \tag{11}$$

$$P\overline{A} + \overline{A}^T P + S + T = -Q < 0 \tag{12}$$

$$0 < R < \frac{(1-\eta)}{\eta} S \tag{13}$$

where $D = PBGA$ and $C = P(BG)^2$.

PROOF: Choose augmented Lyapunov–Krasovskii functionals candidate as follows:

$$\begin{aligned} V(x(t),x(\theta),x(\theta-h),x(t-h)) &= x^T(t)Px(t) + \int_{\theta-h}^{\theta} x^T(\zeta)Rx(\zeta)d\zeta + \int_{\theta}^{t} x^T(\xi)Sx(\xi)d\xi \\ &+ \int_{t-h}^{t} x^T(\varphi)Tx(\varphi)d\varphi \end{aligned} \tag{14}$$

where P, R, S and T are some positive definite symmetric matrices.

The time derivative of (14) along the state trajectory of (9) can be calculated as follows:

$$\begin{aligned} \dot{V} &= x^T(t)P\dot{x}(t) + \dot{x}^T(t)Px(t) + \dot{\theta}(t)\left[x^T(\theta)Rx(\theta) - x^T(\theta-h)Rx(\theta-h)\right] + x^T(t)Sx(t) \\ &\quad - \dot{\theta}(t)x^T(\theta)Sx(\theta) + x^T(t)Tx(t) - x^T(t-h)Tx(t-h) \\ &= x^T(t)(P\overline{A} + \overline{A}^T P)x(t) + 2hx^T(t)PBGAx(\theta) - 2hx^T(t)P(BG)^2x(\theta-h) + \dot{\theta}(t)x^T(\theta)(R-S)x(\theta) \\ &\quad - \dot{\theta}(t)x^T(\theta-h)Rx(\theta-h) + x^T(t)(S+T)x(t) - x^T(t-h)Tx(t-h) \end{aligned} \tag{15}$$

Since $0 < 1-\eta \le \dot{\theta}(t) \le \eta < 1$. Then

$$\begin{aligned} \dot{V} &\le x^T(t)(P\overline{A} + \overline{A}^T P + S + T)x(t) + 2hx^T(t)PBGAx(\theta) - 2hx^T(t)P(BG)^2x(\theta-h) + \eta x^T(\theta)Rx(\theta) \\ &\quad -(1-\eta)x^T(\theta)Sx(\theta) - (1-\eta)x^T(\theta-h)Rx(\theta-h) - x^T(t-h)Tx(t-h) \\ &= \begin{bmatrix} x(t) \\ x(\theta) \\ x(\theta-h) \\ x(t-h) \end{bmatrix}^T \begin{bmatrix} -Q & hD & -hC & 0 \\ hD^T & \eta R-(1-\eta)S & 0 & 0 \\ -hC^T & 0 & -(1-\eta)R & 0 \\ 0 & 0 & 0 & -T \end{bmatrix} \begin{bmatrix} x(t) \\ x(\theta) \\ x(\theta-h) \\ x(t-h) \end{bmatrix} \\ &= z^T(t)Hz(t) < -\lambda_{\min}(H)\|z(t)\|^2 < 0 \end{aligned} \tag{16}$$

where $z(t) = \left[x(t) \;\; x(\theta) \;\; x(\theta-h) \;\; x(t-h)\right]^T$, $D = PBGA$ and $C = P(BG)^2$, if conditions (11), (12) and (13) are satisfied.

Note that matrix H has its own quadratic structure $H = MH_1M^T$, where

$$H_1 = \begin{bmatrix} -Q & 0 & 0 & 0 \\ 0 & \eta R-(1-\eta)S & 0 & 0 \\ 0 & 0 & -(1-\eta)R & 0 \\ 0 & 0 & 0 & -T \end{bmatrix}, \; M = \begin{bmatrix} I & hD\left[\eta R-(1-\eta)S\right]^{-1} & hC\left[(1-\eta)R\right]^{-1} & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix}$$

Since M is a nonsingular and $H_1 < 0$ because its leading principle elements are always negative then $H < 0$. Therefore, condition (11) is feasible. Thus, the transformed time-delay system (9) with known delay is delay-dependent asymptotically stable.

If we consider a case where the time-delay term is unknown but bounded $0 < h < \overline{h}$ then we can solve the following convex optimization problem:

$$\text{OP: maximize h Subject to conditions (11)-(13) and } P, R, S, T > 0 \tag{17}$$

which is a quasi – convex optimization problem. Hence it is possible to compute the maximum upper bound h using efficient convex optimization algorithms by Boyd, Ghaoui, Feron, and Balakishnan [40], etc.

Theorem 1 is proven.

3. ROBUST STABILIZATION OF INPUT–DELAYED SYSTEMS WITH PARAMETER UNCERTAINTIES

The design approach advanced in section 2 can be extended to robust stabilization of input-delayed systems with parameter uncertainties. State equations of this class of systems can be presented as follows:

$$\dot{x}(t) = \left(A + \Delta A(\sigma)\right)x(t) + Bu(t-h)$$
$$u(t) = \phi(t)\,, \quad -h \le t < 0 \tag{18}$$

where in addition to (8) $\Delta A(\sigma)$ are the parameter uncertainties. It is assumed that $\max\left\|\Delta A(\sigma)\right\| \le \alpha$, σ is an uncertain element.

Substituting (3) with (5) into (18) we have:

$$\dot{x}(t) = \bar{A}\,x(t) + \Delta A(\sigma)x(t) + hBGAx(\theta) + hBG\Delta A(\sigma)x(\theta) - h(BG)^2 x(\theta - h) \tag{19}$$

Now, delay–dependent robust stability conditions for transformed time-delayed system with parameter uncertainties can be formulated as follows.

THEOREM 2: Suppose that Assumption 1 holds. Then, the transformed time–delay system with parameter uncertainties (19) driven by linear controller (5) is robustly asymptotically stable, if there are the design parameters G and positive definite symmetric matrices *P*, *R*, *T* and *S* such that the following conditions are satisfied:

$$H = \begin{bmatrix} -Q & hD + \alpha\lambda_{\max}(PBG)I & -hC & 0 \\ hD^T + \alpha\lambda_{\max}(PBG)I & \eta R - (1-\eta)S & 0 & 0 \\ -hC^T & 0 & -(1-\eta)R & 0 \\ 0 & 0 & 0 & -T \end{bmatrix} < 0 \tag{20}$$

$$P\bar{A} + \bar{A}^T P + S + T + \alpha\lambda_{\max}(P)I = -Q < 0 \tag{21}$$

$$0 < R < \frac{(1-\eta)}{\eta}S \tag{22}$$

where $D = PBGA$ and $C = P(BG)^2$.

PROOF: Again consider an extended Lyapunov–Krasovskii functional of the form (14).
The time derivative of (14) along (19) is given by:

$$\begin{aligned}\dot{V} &= x^T(t)(P\bar{A} + \bar{A}^T P)x(t) + 2x^T(t)P\Delta A(\sigma)x(t) + 2hx^T(t)PBGAx(\theta) \\ &+ 2hx^T(t)PBG\Delta A(\sigma)x(\theta) - 2hx^T(t)P(BG)^2 x(\theta - h) + \dot{\theta}(t)x^T(\theta)(R - S)x(\theta) \\ &- \dot{\theta}(t)x^T(\theta - h)Rx(\theta - h) + x^T(t)(S + T)x(t) - x^T(t-h)Tx(t-h)\end{aligned} \tag{23}$$

Since

$$0 < 1-\eta \le \dot{\theta}(t) \le \eta < 1 \tag{24}$$

$$x^T(t)P\Delta A(\sigma)x(t) \le \alpha\lambda_{\max}(P)x^T(t)x(t) \tag{25}$$

$$x^T(t)PBG\Delta A(\sigma)x(\theta) \le \alpha\lambda_{\max}(PBG)x^T(t)x(t) \tag{26}$$

Then (23) can be evaluated as follows:

$$= \begin{bmatrix} x(t) \\ x(\theta) \\ x(\theta-h) \\ x(t-h) \end{bmatrix}^T \begin{bmatrix} -Q & hD+\alpha\lambda_{\max}(PBG)I & -hC & 0 \\ hD^T+\alpha\lambda_{\max}(PBG)I & \eta R-(1-\eta)S & 0 & 0 \\ -hC^T & 0 & -(1-\eta)R & 0 \\ 0 & 0 & 0 & -T \end{bmatrix} \begin{bmatrix} x(t) \\ x(\theta) \\ x(\theta-h) \\ x(t-h) \end{bmatrix}$$

$$= z^T(t)Hz(t) < -\lambda_{\min}(H)\|z(t)\|^2 < 0 \tag{27}$$

where $z(t) = \begin{bmatrix} x(t) & x(\theta) & x(\theta-h) & x(t-h) \end{bmatrix}^T$, if the conditions (20), (21) and (22) are satisfied. Note that matrix H has its own quadratic structure $H = MH_1M^T$, where

$$H_1 = \begin{bmatrix} -Q & 0 & 0 & 0 \\ 0 & \eta R-(1-\eta)S & 0 & 0 \\ 0 & 0 & -(1-\eta)R & 0 \\ 0 & 0 & 0 & -T \end{bmatrix}, M = \begin{bmatrix} I & \bar{D}\left[\eta R-(1-\eta)S\right]^{-1} & hC\left[(1-\eta)R\right]^{-1} & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix}$$

where $\bar{D} = hD + \alpha\lambda_{\max}(PBG)I$. Since M is a nonsingular matrix and $H_1 < 0$ because its leading principle elements are always negative then $H < 0$. Therefore, condition (20) is feasible. Thus, the transformed time-delay system (19) with known delay is delay-dependent asymptotically stable.

If we consider a case where time-delay term is unknown but bounded $0 < h < \bar{h}$ then we can solve the following convex optimization problem:

OP: maximize h Subject to conditions (20)-(22) and $P, R, S, T > 0$ (28)

which is a quasi – convex optimization problem. Hence it is possible to compute the maximum upper bound $\bar{h}$ using efficient convex optimization algorithms by Boyd, Ghaoui, Feron, and Balakishnan [40], etc.

Theorem 2 is proven.

Numerical Example 1: Rocket motor control

Let us consider a liquid monopropellant rocket motor with a pressure feeding system, which is more practical and complex example. This system is not delay-independently stabilizable either. Original complete dynamics model of rocket motor is given by Fiagbedzi and Pearson (1986) [41]. A linearized version of the feeding system and combustion chamber equations assuming non-steady flow is taken from [40] and Moon, Park, Kwon and Lee (2001), [41]:

$$\dot{x}(t) = Ax(t) + A_1x(t-h) + Bu(t) \tag{29}$$

where $x(t)=\left[\phi(t) \quad \mu_1(t) \quad \mu(t) \quad \psi(t)\right]^T$; $\phi(t)$ is the non-dimensional instantaneous pressure in the combustion chamber, $\mu_1(t)$ is the non-dimensional instantaneous mass flow upstream of the capacitance, $\mu(t)$ is the non-dimensional instantaneous mass rate of injected propellant and $\psi(t)$ is the non-dimensional instantaneous pressure at the place in the feeding line. h is the reduced time lag in steady operations, $h \le 1$.

$$u(t)=\frac{(p_0-p_1)}{2\Delta p} \tag{30}$$

is the non-dimensional pressure control input.

$$A=\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & -1 & 1 \\ 0 & 1 & -1 & 0 \end{bmatrix};\ A_1=\begin{bmatrix} -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix};\ B=\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{31}$$

System Equation (29) can be converted to our input-delayed system form as follows. Substituting

$u(t)=-Gx(t)$ into (29) we have

$$\dot{x}(t)=(A-BG)x(t)+A_1x(t-h) \tag{32}$$

Linear controller (5) computational algorithm for input delayed system (1) with stability conditions (11), (12), (13) can be fulfilled by the following MATLAB programming steps:

```
clear; clc;
A      = [0 0 0 0; 0 0 0 -1; -1 0 -1 1; 0 1 -1 0];
A1     = [-1 0 1 0; 0 0 0 0; 0 0 0 0; 0 0 0 0];
B      = [0 0 0 0; 0 1 0 0; 0 0 0 0; 0 0 0 0];
Lamda  = eig(A);
Poles  = [-0.4+1.27i; -0.4-1.27i; -0.21; -.1];
G      = place(A,A1,Poles);
aa     = 6.26;                h      = 0.244;
eta    = 0.005;               R      = 0.5*eye(4);
Q      = 0.9*eye(4);          S      = aa*(eta*R)/(1-eta);
Q1     = eye(4);              T      = Q1 - aa*S - Q;
eQ1    = aa*S + T + Q;        Ahat   = A - (B+A1)*G;
P      = lyap(eQ1,Ahat);      LamdaP = eig(P);
D      = P*A1*G*A;            C      = P*(A1*G*A1*G);
A1*G;                         o0     = eye(4)-eye(4);
H      = [-Q h*D -h*C o0; h*D' eta*R-(1-eta)*S o0 o0;
           -h*C' o0 -(1-eta)*R o0; o0 o0 o0 -T];
Heig   = eig(H)
```

Computational results are

$$eig(A)=\begin{bmatrix} -0.2151+1.3071i \\ -0.2151-1.3071i \\ -0.5698 \\ 0 \end{bmatrix},\ Poles=\begin{bmatrix} -0.4000+1.2700i \\ -0.4000-1.2700i \\ -0.2100 \\ -0.1000 \end{bmatrix},$$

$$G=\begin{bmatrix} -0.0550 & -0.0023 & -0.0340 & 0.3268 \\ 0 & 0 & 0 & 0 \\ 0.0550 & 0.0023 & 0.0340 & -0.3268 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$aa = 6.26$, $h_{\max} = 0.2440$ sec, $\eta = 0.0050$,

$$R = \begin{bmatrix} 0.5000 & 0 & 0 & 0 \\ 0 & 0.5000 & 0 & 0 \\ 0 & 0 & 0.5000 & 0 \\ 0 & 0 & 0 & 0.5000 \end{bmatrix}, \quad S = \begin{bmatrix} 0.0157 & 0 & 0 & 0 \\ 0 & 0.0157 & 0 & 0 \\ 0 & 0 & 0.0157 & 0 \\ 0 & 0 & 0 & 0.0157 \end{bmatrix},$$

$$Q = \begin{bmatrix} 0.9000 & 0 & 0 & 0 \\ 0 & 0.9000 & 0 & 0 \\ 0 & 0 & 0.9000 & 0 \\ 0 & 0 & 0 & 0.9000 \end{bmatrix},$$

$$T = \begin{bmatrix} 0.0015 & 0 & 0 & 0 \\ 0 & 0.0015 & 0 & 0 \\ 0 & 0 & 0.0015 & 0 \\ 0 & 0 & 0 & 0.0015 \end{bmatrix}, \quad Q_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad P = \begin{bmatrix} 0.0550 & 0.0023 & 0.0340 & -0.3268 \\ 0 & 0 & 0 & 0.5000 \\ 0.5000 & 0 & 0.5000 & -0.5000 \\ 0 & -0.5000 & 0.5000 & 0 \end{bmatrix},$$

$$eig(P) = \begin{bmatrix} 0.2000 + 0.6350i \\ 0.2000 - 0.6350i \\ 0.0500 \\ 0.1050 \end{bmatrix}, \quad D = \begin{bmatrix} -0.0037 & -0.0359 & 0.0322 & 0.0035 \\ 0 & 0 & 0 & 0 \\ -0.0340 & -0.3268 & 0.2928 & 0.0317 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$C = \begin{bmatrix} 0.0007 & 0.0000 & 0.0004 & -0.0040 \\ 0 & 0 & 0 & 0 \\ 0.0061 & 0.0003 & 0.0037 & -0.0359 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad A_1 * G = \begin{bmatrix} 0.1100 & 0.0047 & 0.0681 & -0.6536 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$H = \begin{bmatrix}
-0.9000 & 0 & 0 & 0 & -0.0009 & -0.0088 & 0.0079 & 0.0009 & -0.0002 & -0.0000 & -0.0001 & 0.0010 & 0 & 0 & 0 & 0 \\
0 & -0.9000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -0.9000 & 0 & -0.0083 & -0.0797 & 0.0714 & 0.0077 & -0.0015 & -0.0001 & -0.0009 & 0.0088 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -0.9000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-0.0009 & 0 & -0.0083 & 0 & -0.0132 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-0.0088 & 0 & -0.0797 & 0 & 0 & -0.0132 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.0079 & 0 & 0.0714 & 0 & 0 & 0 & -0.0132 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.0009 & 0 & 0.0077 & 0 & 0 & 0 & 0 & -0.0132 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-0.0002 & 0 & -0.0015 & 0 & 0 & 0 & 0 & 0 & -0.4975 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-0.0000 & 0 & -0.0001 & 0 & 0 & 0 & 0 & 0 & 0 & -0.4975 & 0 & 0 & 0 & 0 & 0 & 0 \\
-0.0001 & 0 & -0.0009 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.4975 & 0 & 0 & 0 & 0 & 0 \\
0.0010 & 0 & 0.0088 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.4975 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.0015 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.0015 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.0015 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.0015
\end{bmatrix}$$

$$eig(H) = [-0.9132 \;\; -0.9 \;\; -0.9 \;\; -0.9 \;\; -0.4975 \;\; -0.4975 \;\; -0.4975 \;\; -0.4973 \;\; -0.0132 \;\; -0.0131 \;\; -0.0131 \;\; -0.0015 \;\; -0.0015 \;\; -0.0015 \;\; -0.0015 \;\; -0.0001]$$

4. CONCLUSIONS

In this paper, a new design approach based on Lagrange mean value theorem is used for the first time for the stabilization of multivariable input delayed system by linear controller. The delay–dependent asymptotical stability conditions are derived by using augmented Lyapunov-Krasovskii functionals and formulated in terms of conventional Lyapunov matrix equations and some simple matrix inequalities. Proposed design approach is extended to robust stabilization of multi-variable input delayed systems with unmatched parameter uncertainties. The maximum upper bound of delay size is computed by using simple optimization algorithm. A liquid monopropellant rocket motor with a pressure feeding system is considered as a numerical design example. Design example shows the effectiveness of our proposed design approach.

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